13. UBERALL H., Surface waves in acoustics. In: Physical Acoustics. Principles and Methods, Vol.10, New York-London, Acad. Press, 1973.
14. NUSSENZVEIG H.M., High-frequency scattering by a transparent sphere. Theory of the rainbow and the glory. J. Math. Phys., Vol.10, No.1, 1969.
15. NUSSENZVEIG H.M., The theory of the rainbow, Scient. American, Vol.236, No.4, 1977.
16. FEDORYUK M.V., The Method of Steepest Descent. Moscow, NAUKA, 1977.

Translated by J.J.D.

PMM U.S.S.R.,Vol.47,No.6,pp.767-774,1983
Printed in Great Britain

0021-8928/83 $10.00+0.00

UDC 539.3:534.26

DIFFRACTION OF A PLANE WAVE BY AN INFINITE ELASTIC PLATE STIFFENED BY A DOUBLY PERIODIC SET OF RIGID RIBS*

B. P. BELINSKII

The diffraction of a plane wave by an infinite elastic plate stiffened by a doubly periodic set of rigid ribs of moderate wave dimensions is studied. The problem is reduced to an infinite quasiregular system of linear algebraic equations, and their solution describes the amplitudes of the waves propagating from the plate into the fluid.

The effect of a periodic set of parallel ribs, which stiffen an elastic plate, on its acoustic properties, has been studied in reasonable detail. An exact solution of the problem of the diffraction of a plane wave by such a plate is given in /1/ where the frequency relationships of the reflection and transmission coefficients of a plane wave were also studied and simple approximate formulas were obtained for the limiting cases.

1. We will investigate the diffraction of a plane pressure wave

$$p_0 = \exp(ik((x\cos\varphi_0 + y\sin\varphi_0)\sin\theta_0 - z\cos\theta_0))$$

incident on an infinite plate $\{-\infty < x, y < \infty, z = 0\}$ stiffened by a doubly periodic set of rigid ribs $\{-\infty < x < \infty, y = mb; -\infty < y < \infty, x = na; -\infty < n, m < \infty\}$. The pressure $p(x, y, z)$ in the medium satisfies the Helmholtz equation with the boundary condition at the plate given by

$$D(\Delta_0^2 - k_0^4)\xi(x, y) + [p](z = 0) = 0 \qquad (1.1)$$

$$(x \neq na,\ y \neq mb)$$

$$k_0 = (\rho^\circ\omega^2 H^\circ/D)^{1/4}$$

Here D is the cylindrical rigidity of the plate, $\xi(x, y)$ is its displacement, connected with the pressure by the adhesion condition $\xi(x, y) = p_z(x, y, 0)/(\rho_0\omega^2)$, ρ_0 is the density of the liquid, Δ_0 is the two-dimensional Laplace operator, k_0 is the wave number of the flexural waves in the plate, ρ° is the plate density and H° is its thickness. The symbol $[\varphi](z = 0)$ denotes the jump in the value of the function φ at $z = 0$. The harmonic dependence of the processes on time $\exp(-i\omega t)$ is omitted.

We will first assume that fluid is present on one side of the plate only $(z > 0)$. The case of two-sided contact can be studied in exactly the same manner. We shall therefore only refer to it at the stage of numerical analysis and interpretation of the results. The boundary contact conditions (BCC) appear when the bending and torsional oscillations of the ribs and their rigid coupling to the plate carrying them are taken into account /2/

$$-D[\xi_{xxx} + (2-\sigma)\xi_{xyy}](x = na) = -i\omega Z_{11}\xi \qquad (1.2)$$

$$D[\xi_{xx} + \sigma\xi_{yy}](x = na) = -i\omega Z_{12}\xi_x$$

$$(x = na,\ y \neq mb)$$

$$-D[\xi_{yyy} + (2-\sigma)\xi_{yxx}](y = mb) = -i\omega Z_{21}\xi$$

$$D[\xi_{yy} + \sigma\xi_{xx}](y = mb) = -i\omega Z_{22}\xi_y\ (y = mb,\ x \neq na)$$

Here σ is Poisson's ratio of the plate, and the operators $Z_{p1}, Z_{p2}\ (p = 1, 2)$ are the force and momentum impedances of the ribs respectively.

*Prikl.Matem.Mekhan.,47,6,962-971,1983

Unlike the well-known boundary contact problems /3—5/, the three-dimensional boundary value problem described here can be called a second-order boundary contact problem since, in addition to the boundary condition (1.1) and BCC (1.2), we must also formulate the BCC at the points of intersection of the ribs ($x = na, y = mb$). Below we shall describe the class of admissible second-order BCC. The specific analysis will be performed for two parameters

$$\xi = \xi_x = \xi_y = 0 \tag{1.3}$$

$$F = M_1 = M_2 = 0 \tag{1.4}$$

$$(F = [E_2 I_2 \xi_{xxx}]\,(x = na) + [E_1 I_1 \xi_{yyy}]\,(y = mb)$$

$$M_1 = -[E_1 I_1 \xi_{yy}]\ \ (y = mb),\ M_2 = -[E_2 I_2 \xi_{xx}]\ \ (x = na))$$

In the first case the nodes are assumed to be rigidly fixed, and free in the second case. Here F denotes the concentrated force appearing at the node, M_1 and M_2 are concentrated moments, and the quantities $E_p, I_p\ (p = 1, 2)$ denote Young's modulus and the moments of inertia of the corresponding ribs. Finally, the diffuse field $q = p - p_0$ must satisfy the principle of limit absorption. An analogous problem was first studied in /6/, without offering a mathematical justification for the numerical algorithm employed, and the problem of second-order BCC was not formulated. We shall show that this implies that it was case (1.4) of free nodes that was studied.

2. In view of the fact that the incident wave is almost periodic, we shall seek the diffuse field in the form of an almost-periodic function

$$q\,(x + na,\ y + mb,\ z) = q\,(x,\ y,\ z) \exp\,(in\alpha + im\beta) \tag{2.1}$$

$$\alpha = ka \cos \varphi_0 \sin \theta_0,\ \ \beta = kb \sin \varphi_0 \sin \theta_0$$

and consider the boundary value problem in the fundamental period $\Omega_0 = \{0 < x < a,\ 0 < y < b,\ 0 < z < \infty\}$. To justify the scheme, which is based on the application of the principle of limiting absorption, we must establish the uniqueness of the solution of the homogeneous boundary value problem $(p_0 = 0)$ when there is absorption in the medium $(\mathrm{Im}\, k > 0)$. The solution is sought in the class of functions with a finite norm in $L_2\,(\Omega_0)$. We shall make use of the identity following from Green's second formula for the Laplace operator

$$-\,\mathrm{Im}\, k^2 \,\| q \|^2_{L_2(\Omega)} = \mathrm{Im} \int\limits_{\partial\Omega} \frac{\partial q}{\partial n}\, \bar{q}\, dS, \quad \Omega = S_1 \times (0 < z < \infty)$$

Here S_1 denotes a certain translation of the fundamental period of the plate, such that only one node of the mesh (0,0) lies within it, and the bar denotes a complex conjugate. In transforming the right-hand side describing the energy flux across the boundary $\partial\Omega$, we use the Green's second formula for a plate with ribs /7/. As a result we obtain the following indentity:

$$-\,\frac{\mathrm{Im}\, k^2}{2\rho_0\omega} \| q \|^2_{L_2(\Omega)} = \frac{\omega}{2}\, \mathrm{Im}\, \{\bar{\xi} F + \bar{\xi}_y M_1 + \bar{\xi}_x M_2\}\,(0,\, 0) \tag{2.2}$$

We shall call the second-order BCC admissible if the right-hand side of this identity is non-negative. For example, BCC of the form $(F, M_1, M_2)^t = Z\,(\xi,\ \xi_y,\ \xi_x)^t$ (t denotes transposition) with the impedance matrix Z, the eigenvalues of which have non-negative imaginary parts, will be admissible. We can consider the BCC (1.3) and (1.4) as special cases as $Z \to \infty$ and $Z \to 0$ respectively. It is obvious that $q = 0$ for the admissible BCC, and this implies that the solution is unique.

Henceforth, it will be convenient to represent the diffuse field by a double Fourier series with unknown amplitudes $A_{n,\,m}$

$$q\,(x, y, z) = \frac{-1}{ab} \sum \sum A_{nm} \exp\,(i\lambda_n x + i\mu_m y - \gamma_{nm} z) \tag{2.3}$$

$$\left(\lambda_n = \frac{2\pi n + \alpha}{a},\ \mu_m = \frac{2\pi m + \beta}{b},\ \gamma_{nm} = (\lambda_n^2 + \mu_m^2 - k^2)^{1/2}, \qquad \mathrm{Re}\, \gamma_{n,\,m} \geqslant 0\right)$$

Here and henceforth the fact that there are no limits on the summation signs means that the summation is carried out over all integer values of n and m. When there is no absorption $(\mathrm{Im}\, k = 0)$ the energy identity has the form

$$\frac{ab}{2\rho_0\omega} \sum \sum |A_{n,\,m}|^2\, \mathrm{Re}\, \sqrt{k^2 - \lambda_n^2 - \mu_m^2} = 0 \tag{2.4}$$

and the summation need only to be carried out over the waves propagating upwards for which $\lambda_n^2 + \mu_m^2 < k^2$. Under the conditions of the homogeneous problem the amplitude of these waves will thus be zero. At certain frequencies however, solutions of the homogeneous boundary value problem of the wave type (2.3) moving along the construction and decreasing exponentially with distance from it, may appear.

3. To find the diffraction component of the field q we shall separate from the total field not only the incident wave, but also the wave p_1 reflected from the homogeneous plate

$$p = p_0 + p_1 + q,\ p_1 = R \exp (i\ (\alpha x/a + \beta y/b + kz \cos\theta_0))$$
$$R = R_-/R_+,\ R_\pm = (k^4 \sin^4\theta_0 - k_0^4)\ ik \cos\theta_0 \pm \nu,\ \ \nu = \rho_0\omega^2/D$$

Following the accepted methods of solving boundary contact problems, we shall rewrite the boundary condition (1.1) treating it as inhomogeneous

$$\begin{aligned} &D(\Delta_0^2 - k_0^4)\xi(x, y) + q(x, y, 0) = \\ &\frac{1}{\nu}\sum_n e^{in\alpha}(\delta(x-na)B_1(y) + \delta'(x-na)C_1(y)) + \\ &\frac{1}{\nu}\sum_m e^{im\beta}(\delta(y-mb)B_2(x) + \delta'(y-mb)C_2(x)) \\ &(-\infty < x, y < \infty) \end{aligned} \tag{3.1}$$

Here and henceforth the symbol n or m appearing under the summation sign will denote summation over all integer values of n or m respectively. The unknown functions B_1, B_2, C_1, C_2 are analogs of the boundary contact constants appearing in plane problems of acoustics /3–5/. Substituting into (3.1) the field q in the form of the series (2.3) and using the condition of adhesion, we obtain

$$\begin{aligned} A_{n,m} &= (p_{1n} + i\,\mu_m q_{1n} + p_{2m} + i\lambda_n q_{2m})/L_{n,m} \\ L_{n,m} &= ((\lambda_n^2 + \mu_m^2)^2 - k_0^4)\,\gamma_{nm} - \nu \end{aligned} \tag{3.2}$$

Here $p_{1n}, q_{1n}, p_{2m}, q_{2m}$ are the Fourier coefficients of the functions $B_2(x), C_2(x), B_1(y), C_1(y)$ respectively. The displacement field has the form

$$\xi(x, y) = \frac{1}{\rho_0\omega^2 ab}\sum\sum\frac{\gamma_{n,m}}{L_{n,m}}(p_{1n} + i\mu_m q_{1n} + p_{2m} + i\lambda_n q_{2m})\exp(i(\lambda_n x + \mu_m y)) \tag{3.3}$$

The continuity of displacements and angles of rotation of the plate when passing across the ribs is ensured by requiring that the following asymptotic estimates hold for the unknown $p_{1n}, q_{1n}, p_{2m}, q_{2m}$:

$$p_{1n}, p_{2m} = O(1),\ q_{1n} = O(1/n),\ q_{2m} = O(1/m)\ (n, m \to \infty)$$

It can be confirmed that the above estimates guarantee the finiteness of the potential energy of a single period of the plate. The BCC (1.2) contain the discontinuities of high-order derivatives of the displacement field ξ. It can be shown that a discontinuity in any derivative of up to the third order in ξ is the same as the discontinuity for the field ξ_1 derived from the field ξ by discarding from the symbol of the boundary operator $L_{n,m}$ its dynamic part $(\nu \to 0)$

$$\xi_1(x, y) = \frac{1}{\rho_0\omega^2 ab}\sum\sum\frac{1}{(\lambda_n^2 + \mu_m^2)^2}(p_{1n} + i\mu_m q_{1n} + p_{2m} + i\lambda_n q_{2m})\exp(i(\lambda_n x + \mu_m y)) \tag{3.4}$$

Moreover, since the discontinuities in the derivatives are connected with the non-uniform convergence of series (3.4), it follows that we can delete from it any finite number of terms.

Below we shall find the following identity useful, the validity of which can be confirmed using the Parseval equation:

$$\sum_n \frac{\exp(i\lambda_n x)}{(\lambda_n^2 + \mu_m^2)^2} = \frac{a}{4|\mu_m|^3}\sum_n \exp(-in\alpha - |\mu_m||x + na|)(1 + |\mu_m||x + na|) \tag{3.5}$$

The identity enables us to confine ourselves to terms with $n = 0$ when computing the necessary jumps in the derivatives on the line $x = 0$. Finally we have, at $y \neq 0$,

$$\begin{aligned} [\xi_{xx}](x=0) &= \frac{1}{\rho_0\omega^2 b}\sum_m q_{2m}\exp(i\mu_m y) \\ [\xi_{xxx}](x=0) &= \frac{1}{\rho_0\omega^2 b}\sum_m p_{2m}\exp(i\mu_m y) \end{aligned} \tag{3.6}$$

The BCC (1.2) are formulated outside the nodes $(x = na,\ y = mb)$. At the nodes we must take into account the concentrated reactions, and this can be done by introducing into (1.2) delta functions with unknown multipliers C_s $(1 \leqslant s \leqslant 4)$. Let us write the first two transformed equations (1.2)

$$-D[\xi_{xxx}](x=na)+\frac{C_1}{\nu}\sum_m \delta(y-mb)\exp(im\beta)= \tag{3.7}$$
$$-i\omega Z_{11}(\xi+\xi_+)$$
$$D[\xi_{xx}](x=na)-\frac{C_2}{\nu}\sum_m \delta(y-mb)\exp(im\beta)=$$
$$-i\omega Z_{12}(\xi_x+\xi_{+x})\quad (x=na, -\infty<y<\infty)$$
$$\xi_+=ik\cos\theta_0\,\frac{R-1}{\rho_0\omega^2}\exp\left(i\left(\alpha\frac{x}{a}+\beta\frac{y}{b}\right)\right)$$

Here ξ_+ is the plate displacement corresponding to the geometrical part of the field. Explicit expressions for the rib impedances parallel to the y-axis /2/ have the form

$$-i\omega Z_{11}=E_1I_1\partial^4/\partial y^4-\rho_1b_1H_1\omega^2,\quad -i\omega Z_{12}=-K_1\partial^2/\partial y^2-\rho_1I_1\omega^2$$

From now on ρ_1, b_1, H_1 will denote the density, height and thickness of the ribs, and K_1 and I_1 their torsional rigidity and the moment of inertia of the cross section. The corresponding quantities for the ribs parallel to the x-axis will be given the subscript 2.

Let us introduce the symbols for the impedance operators, i.e. their Fourier transforms normalized to the rigidity of the plate

$$\Omega_{1n}=(E_2I_2\lambda_n^4-\rho_2b_2H_2\omega^2)/(bD),\ W_{1n}=(K_2\lambda_n^2-\rho_2I_2\omega^2)/(bD) \tag{3.8}$$
$$\Omega_{2m}=(E_1I_1\mu_m^4-\rho_1b_1H_1\omega^2)/(aD),\ W_{2m}=$$
$$(K_1\mu_m^2-\rho_1I_1\omega^2)\ /\ (aD)$$

Substituting into BCC (3.7) the series for the displacements (3.3) and taking into account the expressions for the jumps (3.6), we arrive at the following system of linear algebraic equations:

$$p_{1n}+\Omega_{1n}\left(\sum_m\rho_{nm}p_{2m}+i\lambda_n\sum_m\rho_{nm}q_{2m}+J_n^\circ p_{1n}+J_n^1q_{1n}\right)= \tag{3.9}$$
$$-\Omega_{10}d\delta_n^\circ+C_3$$
$$q_{1n}-W_{1n}\left(\sum_m\rho_{nm}i\mu_mp_{2m}+i\lambda_n\sum_m\rho_{nm}i\mu_mq_{2m}+J_n^1p_{1n}+J_n^2q_{1n}\right)=$$
$$W_{10}i\mu_0d\delta_n^\circ+C_4$$
$$p_{2m}+\Omega_{2m}\left(\sum_n\rho_{nm}p_{1n}+i\mu_m\sum_n\rho_{nm}q_{1n}+I_m^\circ p_{2m}+I_m^1q_{2m}\right)=$$
$$-\Omega_{20}d\delta_m^\circ+C_1$$
$$q_{2m}-W_{2m}\left(\sum_n\rho_{nm}p_{1n}i\lambda_n+i\mu_m\sum_n\rho_{nm}i\lambda_nq_{1n}+I_m^1p_{2m}+I_m^2q_{2m}\right)=$$
$$W_{20}i\lambda_0\,d\delta_m^\circ+C_2$$
$$(\rho_{nm}=\gamma_{n,m}/L_{n,m},\ d=ab(R-1)ik\cos\theta_0,\ -\infty<n,m<\infty)$$
$$\left(I_m^s=\sum_n\rho_{nm}(i\lambda_n)^s,\quad J_n^s=\sum_m\rho_{nm}(i\mu_m)^s;\ s=0,1,2\right)$$

where the constants C_s are found from the BCC (1.3) or (1.4). We shall deal with the case of damped nodes only. Let us divide the first equation of (3.9) by Ω_{1n} and carry out the summation over all n. We obtain

$$\sum_n p_{1n}/\Omega_{1n}+S=C_3\sum_n 1/\Omega_{1n}$$
$$\left(S=\sum_n J_n^\circ p_{1n}+\sum_n J_n^1q_{1n}+\sum_m I_m^\circ p_{2m}+\sum_m I_m^1q_{2m}+d\right)$$

At the same time, the condition of clamping the node $\xi(0,0)+\xi_+(0,0)=0$ and representation (3.3) together imply that $S=0$, and as a result we obtain C_3. The final system of linear algebraic equations takes the form

$$U_n\mathbf{p}_n+\sum_{m\neq n}V_{nm}\mathbf{p}_m-\sum_m T_{nm}\mathbf{p}_m=\mathbf{p}_n^\circ\quad(-\infty<n<\infty) \tag{3.10}$$

where

$$U_n=\begin{Vmatrix}\tau_{1n}+J_n^\circ & J_n^1 & \rho_{nn} & i\lambda_n\rho_{nn}\\ -J_n^1 & \tau_{2n}-J_n^2 & -i\mu_n\rho_{nn} & \lambda_n\mu_n\rho_{nn}\\ \rho_{nn} & i\mu_n\rho_{nn} & \eta_{1n}+I_n^\circ & I_n^1\\ -i\lambda_n\rho_{nn} & \lambda_n\mu_n\rho_{nn} & -I_n^1 & \eta_{2n}-I_n^2\end{Vmatrix}$$

$$V_{nm}=\begin{Vmatrix}0 & \rho_{nm}V_{nm}^1\\ \rho_{mn}V_{nm}^2 & 0\end{Vmatrix},\quad V_{nm}^1=\begin{Vmatrix}1 & i\lambda_n\\ -i\mu_m & \lambda_n\mu_m\end{Vmatrix}$$

$$V_{nm}^2 = \begin{Vmatrix} 1 & i\mu_n \\ -i\lambda_m & \lambda_m\mu_n \end{Vmatrix}, \quad \tau_{1n} = 1/\Omega_{1n}, \quad \tau_{2n} = 1/W_{1n}$$
$$\eta_{1m} = 1/\Omega_{2m}, \quad \eta_{2m} = 1/W_{2m}$$
$$\mathbf{p}_n = (p_{1n}, q_{1n}, p_{2n}, q_{2n})^t, \quad \mathbf{p}_n^\circ = d(-1, -i\mu_0, -1, -i\lambda_0)^t \delta_n^\circ$$
$$T_{nm} = \operatorname{diag}(\tau_{1n}\tau_{1m}/\tau_1, \ \tau_{2n}\tau_{2m}/\tau_2, \quad \eta_{1n}\eta_{1m}/\eta_1, \ \eta_{2n}\eta_{2m}/\eta_2)$$
$$\tau_s = \sum_n \tau_{sn}, \quad \eta_s = \sum_n \eta_{sn} \quad (-\infty < n, m < \infty)$$

4. Next we shall study the problems of normal incidence of a plane wave on a plate fitted with a square mesh of identical ribs ($b = a$, $\cos\theta_0 = 1$). System (3.10) then takes the form ($n \geqslant 0$)

$$u_n p_{1n} + \sum_{m\geqslant 0}{}' \varepsilon_m \rho_{nm} p_{1m} - \frac{\tau_{1n}}{\tau_1} \sum_{m\geqslant 0} \varepsilon_m \tau_{1m} p_{1m} = -d\delta_n^\circ \tag{4.1}$$
$$u_n = \tau_{1n} + J_n^\circ + \varepsilon_n \rho_{nn}, \quad \varepsilon_n = 2 - \delta_n^\circ$$

Here and below a prime on the summation sign denotes that there is no term with $m = n$. The complete pressure field is expressed by the soluition of the system p_n

$$p = \exp(-ikz) + R\exp(ikz) - \frac{1}{a^2} \sum \sum \frac{1}{L_{n,m}} (p_n + p_m) \times \exp(i(\lambda_n x + \lambda_m y) - \gamma_{nm} z) \tag{4.2}$$

In particular, the expression for the reflection coefficient of the principal wave is

$$K_0 = R - \frac{2}{a^2} \frac{p_{10}}{L_{0,0}} \tag{4.3}$$

Similar arguments for the case of free nodes yield the following system of equations:

$$u_n p_{1n} + \sum_{m\geqslant 0}{}' \varepsilon_m \rho_{nm} p_{1m} = -d\delta_n^\circ \quad (n \geqslant 0) \tag{4.4}$$

which agrees with the system from /6/. This suggests that the arguments used in that paper imply indirectly the use of BCC (1.4).

Let us use (4.4) to see whether the reduction method can be applied to infinite systems. We divide every equation by u_n and estimate the sum of s_n moduli of the elements of the n-th row of the resulting matrix. Remembering that ρ_{nm} and u_n are positive, we find for large numbers that

$$s_n = \frac{1}{u_n} \sum_{m\geqslant 0}{}' \varepsilon_m \rho_{nm} = \frac{1}{u_n} \Big(\sum_m \rho_{nm} - 2\rho_{nn} \Big) = \frac{1}{u_n} (J_n^\circ - 2\rho_{nn})$$

The asymptotic of J_n° with respect to the subscript can be obtained by comparing this series with its value at zero frequency ω. The latter series can be summed explicitly to yield

$$J_n^\circ = \frac{\pi}{2n^3} \left(\frac{a}{2\pi}\right)^4 (1 + o(1)), \quad u_n = J_n^\circ + O(n^{-4})$$

Taking into account the obvious asymptotic form

$$\tau_{1n} \sim c_0/(2\pi n/a)^4, \ c_0 = aD/(E_1 I_1)$$

we obtain

$$s_n = 1 - s_n', \quad s_n' = \frac{\tau_{1n} + 4\rho_{nn}}{\tau_{1n} + J_n^\circ + 2\rho_{nn}} \sim \frac{2(c_0 + 1)}{\pi n}$$

so that $s_n < 1$ for fairly large n. From the well-known theorems on infinite algebraic systems /8/ we infer that the reduction method can be used when the free terms are estimated using the quantitites Ks_n', where K is any constant. The last estimate is obvious.

We shall also show that the proof of the applicability of the reduction method given in /6/ is based on the hypothesis that the Hilbert-Schmidt norm of the corresponding matrix is finite. Incidentally, the double series with the general term $|\rho_{nm}/u_n|^2$ diverges. We note that the present investigation of the infinite system (4.1), (4.4) is similar to that carried out in /8, 9/ in connection with the problem of the flexure of a rectangular plate with rigidly clamped edges. The systems in question become those of /8, 9/ as $\omega \to 0$, provided that the impedances $Z_{p1}, Z_{p2} \to \infty$.

5. Let us investigate the dynamic behaviour, with respect to the parameters of the problem, of the reflection coefficient K_0 of the principle wave given by (4.3). We will consider the case of free nodes, and restrict ourselves first to the diagonal approximation in system (4.4), i.e. $p_{1n} = -d\delta_n^\circ/u_0$. We have

$$K_0 = \frac{ik\varkappa^4 + \nu}{ik\varkappa^4 - \nu}, \quad \varkappa^4 = k_0^4 - \frac{2}{B}, \quad B = \tau_{10} + 2\sum_{n\geqslant 1}\rho_{n0} \tag{5.1}$$

The approximate formula obtained for the reflection coefficient is superficially analogous to the formula for a homogeneous plate, differing only in the fact that the wave number k_0 is replaced by a derived quantity $\varkappa$ expressing the effect of the stiffeners. Similarly, in the problem of a plane wave passing through a plate we have the following expression for the reflection coefficient K_1 and transmission coefficient T of the principal wave:

$$K_1 = \frac{ik\varkappa^4}{ik\varkappa^4 - 2\nu}, \quad T = 1 - K_1 = \frac{-2\nu}{ik\varkappa^4 - 2\nu} \tag{5.2}$$

and in computing B we should take the symbol for the plate operator in the form

$$L_{n,m} = ((\lambda_n^2 + \mu_m^2)^2 - k_0^4)\,\gamma_{nm} - 2\nu$$

The approximate formula (5.1) has a simple interpretation. The quantity B is real up to the first "cut-off frequency" $(ka < 2\pi)$, and of all the waves shown in the field (2.3), the only wave still propagating is the null wave $(n = m = 0)$. Therefore $|K_0| = 1$ and the only parameter that changes with the frequency is the phase of the reflected wave. Further, with the same parameters of the problem we find that when $B = \infty$, $\varkappa = k_0$ and the plate behaves as a homogeneous plate with its acoustic field unaffected by the ribs. It is clear that this case occurs when $\lambda a = 2\pi s$, where λ is the wave number of the flexural waves in the plate-fluid system, and s is an integer. When $B = 0$, we have $\varkappa = \infty$, $K_0 = 1$ and the plate behaves as a perfectly rigid surface. Finally, when $B = 2/k_0^4$ we have $\varkappa = 0$, $K_0 = -1$ and the plate becomes perfectly plastic. Similarly, in the problem of wave transmission the value $B = 0$ results in total reflection, and $B = 2/k_0^4$ in total transmission of the incident wave. Thus using the approximate formulas (5.1), (5.2) we can describe all characteristic values of the reflection and transmission coefficients at the frequencies below the first cut-off frequency.

At low frequencies we can retain in the expression for B only the term $\tau_{10} = -aD/(\rho_1 b_1 H_1 \omega^2)$. In this case the formulas for the reflection coefficients will become

$$K_0 = \frac{ik(m + 2m_1) + \rho_0 a^2}{ik(m + 2m_1) - \rho_0 a^2}, \quad K_1 = \frac{ik(m + 2m_1)}{ik(m + 2m_1) - 2\rho_0 a^2} \tag{5.3}$$

$$m = a^2\rho^\circ H^\circ, \quad m_1 = a\rho_1 b_1 H_1$$

The quantities m and $2m_1$ can be regarded as the mass of a single period of the plate and the mass of the ribs per period respectively. Formula (5.3) is known as the mass law /1/ for a plate with a singly periodic set of ribs.

In the case of clamped nodes the diagonal approximation cannot be used in (4.1), and such simple formulas cannot be obtained. We shall establish that, irrespective of the form of BCC, we can write the reflection coefficient of the second kind in the form (5.1) (or (5.2) for an appropriate value of B). We shall write (4.1) and (4.4) uniformly as $(n \geqslant 0)$

$$w_n p_{1n} + \sum_{m\geqslant 0}{}' v_{nm} p_{1m} = -d\delta_n^\circ \tag{5.4}$$

$$w_n = u_n - \eta\frac{\tau_{1n}}{\tau_1}\varepsilon_n, \quad v_{nm} = \varepsilon_m\left(\rho_{nm} - \eta\frac{\tau_{1n}^2\tau_{1m}}{\tau_1}\right)$$

($\eta = 1$ in the case of clamped nodes and $\eta = 0$ for free nodes).

We separate in (5.4) the equation with $n = 0$ and put $p_{1n} = -p_{10}X_n$ with the new unknown $X_n\ (n \geqslant 1)$. For the latter we have the following infinite system of equations:

$$w_n X_n + \sum_{m\geqslant 1}{}' v_{nm} X_m = v_{n0} \tag{5.5}$$

We introduce the solution matrix $\|\mu_{nm}\|$ for this system, so that

$$X_n = \sum_{m\geqslant 1} \mu_{nm} v_{m0}$$

Then the first equation of system (5.4) yields

$$p_{10} = -d\left(w_0 - \sum_{m\geqslant 1} v_{0m} \sum_{k\geqslant 1} \mu_{mk} v_{k0}\right)$$

Let us now introduce the effective quantity B_{0*} for the case of one-sided contact between the plate and the liquid

$$B_{0*} = \tau_{10} - \eta\frac{\tau_{10}^2}{\tau_1} + 2\sum_{n\geqslant 1}\rho_{n0} - \sum_{m\geqslant 1} v_{0m} \sum_{k\geqslant 1} \mu_{mk} v_{k0} \tag{5.6}$$

Replacing the parameter ν by 2ν in the symbol for the plate operator we obtain the effective quantity B_{1*} for the case of two-sided contact. Then the exact expression for the reflection coefficient (4.3) becomes

$$K_{0*} = \frac{ik\varkappa_{0*}^4 + \nu}{ik\varkappa_{0*}^4 - \nu}, \quad \varkappa_{0*}^4 = k_0^4 - \frac{2}{B_{0*}} \tag{5.7}$$

$$K_{1*} = \frac{ik\varkappa_{1*}^4}{ik\varkappa_{1*}^4 - 2\nu}, \quad \varkappa_{1*}^4 = k_0^4 - \frac{2}{B_{1*}} \tag{5.8}$$

for one-sided and two-sided contact respectively. In the case of clamped nodes we have a low-frequency asymptotic form

$$B_* \sim 2\left(\frac{a}{2\pi}\right)^4 \sum_{n\geqslant 1} \frac{1}{n^4}(1 + \psi - (1-\psi)^2 \xi_n), \quad \psi = \frac{3aD}{b_1^3 H_1 E_1}$$

where ξ_n $(n \geqslant 1)$ is the solution of the system of equations with coefficients independent of the frequency

$$\left(\frac{3}{2} + \psi + 2n^4 \sum_{m\geqslant 1} \frac{1}{(n^2+m^2)^2}\right)\xi_n + 2n^4 \sum_{m\geqslant 1}{}' \frac{1}{(n^2+m^2)^2}\xi_m = 1$$

When $\psi = 0$, the system is identical with the system given in /9/.

6. Certain energy identities are of use in checking the numerical calculations. Applying the formulas of Sect.2 to the total field we obtain, in the same way as (2.4),

$$\sum\sum |s_{nm}|^2 \operatorname{Re} \sqrt{k^2 - \lambda_n^2 - \mu_m^2} = \sqrt{k^2 - \lambda_0^2 - \mu_0^2} \tag{6.1}$$

Here the summation is carried out over the propagating waves represented in the scattered field

$$p_1 + q = \sum\sum s_{nm} \exp(i(\lambda_n x + \mu_m y) - \gamma_{nm} z) \tag{6.2}$$

Let us consider the frequencies below the first cut-off frequency and separate from the amplitude s_{00} the diffraction component $r = s_{00} - R$. Then (6.1) can be rewritten in one of the following forms:

$$|s_{00}| = |r + R| = 1, \quad |r|^2 = -2\operatorname{Re}(\bar{R}r) \tag{6.3}$$

In the case of two-sided contact between the plate and the liquid, we introduce an additional transverse field $(z < 0)$

$$q' = \sum\sum s_{nm}' \exp(i(\lambda_n x + \mu_m y) + \gamma_{nm} z) \tag{6.4}$$

and separate the diffraction component $t = s_{00}' - T$ (T is the transmission coefficient for the homogeneous plate) from the amplitude of the principal wave. In this case (6.1) will take the form

$$\sum\sum (|s_{nm}|^2 + |s_{nm}'|^2) \operatorname{Re} \sqrt{k^2 - \lambda_n^2 - \mu_m^2} = \sqrt{k^2 - \lambda_0^2 - \mu_0^2} \tag{6.5}$$

and identity (6.3) will be replaced by

$$\begin{gathered} |s_{00}|^2 + |s_{00}'|^2 = |r + R|^2 + |t + T|^2 = 1 \\ |r|^2 + |t|^2 = -2\operatorname{Re}(\bar{R}r + \bar{T}t) \end{gathered} \tag{6.6}$$

Relations (6.3) and (6.6) are analogs of the optical theorem for the model in question /10/.

As an example of the application of (6.5) we shall obtain an estimate for the amount of energy transferred from the incident wave to all other waves. We write the amplitude s_{00} in complex form: $s_{00} = u + iv$, whereupon we have $s_{00}' = 1 - u - iv$ and a whole sequence of obvious relationships

$$\begin{gathered} \sum_{n^2+m^2>0}\sum (|s_{nm}|^2 + |s_{nm}'|^2) \operatorname{Re} \sqrt{k^2 - \lambda_n^2 - \mu_m^2} = \\ \sqrt{k^2 - \lambda_0^2 - \mu_0^2}(1 - |s_{00}|^2 - |s_{00}'|^2) = \\ \sqrt{k^2 - \lambda_0^2 - \mu_0^2}(2u - 2u^2 - 2v^2) \leqslant \frac{1}{2}\sqrt{k^2 - \lambda_0^2 - \mu_0^2} \end{gathered}$$

Thus the proportion of the energy transferred from the incident wave to all other waves does not exceed 0.5.

Let us now consider a numerical investigation of the dynamics of the reflection coefficient K of the principal wave, with respect to the frequency. We used the exact formula (4.3), by solving the infinite system (5.4) for a steel plate of thickness $H^\circ = 4$ cm., with steel reinforcing ribs of thickness $H_1 = 3$ cm. and height $b_1 = 20$ cm., spaced a distance $a = 60$ cm., for the case of two-sided contact between the plate and water. Fig.1 refers to the free nodes, and Fig.2 to the clamped nodes. The relations connecting the modulus of the reflection coefficient $M = |K|$ and the phase $F = \arg K$ with the frequency are shown. For comparison, the dashed lines depict the same relationship for a hypothetical rib density exceeding that of steel by a factor of 10, and the dot-dash lines refer to the homogeneous plate. The initial segments of the curves in Fig.1 obey the mass law (5.3) up to a frequency of approximately 0.6 kHz. As the frequency increases, the reflection coefficient oscillates about the values corresponding to a homogeneous plate, and narrow transmission zones appear in which the value of the reflection coefficient falls sharply. An increase in the mass of the ribs leads to reduction in the frequencies at which the reflection coefficients undergo sharp changes; this is related to the effective increase in the wave number of the construction. As the spacing between the reinforcing ribs increases, the distance separating the curves constructed for different rib densities decreases and they tend asymptotically to the curves for a homogeneous plate. When $f > 3.5$ kHz (where one and half wavelengths in the liquid can fit between the ribs), the influence of the ribs can be neglected.

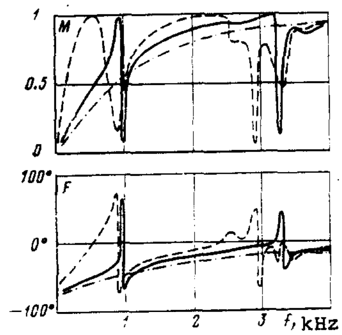

Fig.1

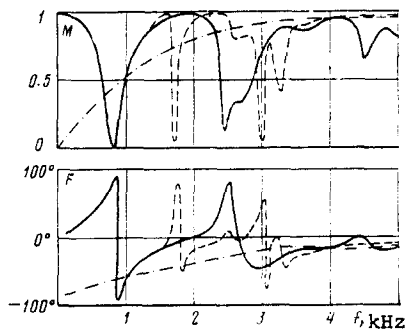

Fig.2

In the case of clamped nodes (Fig.2) the reflection at low frequencies ($f < 0{,}4$ kHz) is almost complete. Below 1.4 kHz the reflection coefficient is independent of the mass of the ribs, the latter behaving as if they were infinitely heavy. The reflection coefficient reaches values corresponding to a homogeneous plate more slowly in the case of clamped nodes than of the free nodes, On the whole, the conditions at the nodes fixed with help of the second-order BCC exert a substantial influence on the acoustic properties of the plate, especially at low and middle frequencies.

REFERENCES

1. SHENDEROV E.L., Wave Problems in Hydroacoustics. Leningrad, Sudostroenie, 1972.
2. BELINSKII B.P., VESHEV V.A., KLYUKIN I.I. and KOUZOV D.P., On the influence of the ribs on the propagation of flexural waves in a plate of finite width. Izv. Akad. Nauk SSSR, MTT, No.5, 1977.
3. KOUZOV D.P., Diffraction of a cylindrical hydroacoustic wave at the join of two semi-infinite plates. PMM Vol.33, No.2, 1968.
4. BELINSKII B.P., KOUZOV D.P. and CHEL'TSOVA V.D., On acoustic wave diffraction by plates connected at a right angle. PMM Vol.37, No.2, 1973.
5. KOUZOV D.P., On the acoustic field of a point source in a rectangular space bounded by thin elastic walls. PMM Vol.43, No.2, 1979.
6. MACE B.R., Sound radiation from fluid loaded orthogonally stiffened plates. J. Sound and Vibrat., Vol.79, No.3, 1981.
7. BELINSKII B.P., On the uniqueness of the solution of stationary problems of acoustics for reinforced plates. Zap. nauch. seminarov LOMI, Vol.104, No.11, 1981.
8. KANTOROVICH L.V. and KRYLOV V.I., Approximate Methods of Higher Analysis. Interscience Publishers, N.Y. and Groningen, 1964.
9. DAREVSKII V.M. and SHARINOV I.L., Novel solution of the problem of the bending of rectangular plate clamped at the edges. In book: Uspekhi mekhaniki deformiruemykh sred. Moscow, NAUKA, 1975.
10. BELINSKII B.P. and KOUZOV D.P., An optical theorem for a plate-liquid system. Akust. zh., Vol.26, No.1, 1980.

Translated by L.K.